Optimal patent life vs optimal patentability standards*

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This note considers a new dimension in optimal patent design by treating strategic roles as a policy instrument. It is shown that, contrary to established wisdom, welfare is not always maximized when the Patent Office optimizes over patent life and plays the role of Stackelberg leader. When innovations are not 'difficult', leadership is irrelevant and setting a (high) minimum patentability standard is more valuable than setting patent life.

1. Introduction

The standard theory of optimal patent life, as formulated by Nordhaus (1969) and Scherer (1972), has recently been extended to include a compulsory royalty rate [Tandon (1982)] product patents [Waterson (1990)] and patent 'breadth' [Gilbert and Shapiro (1990), Klemperer (1990)]. This note considers another dimension of the patent system, namely the game-theoretic implications of the optimal determination of patent life vs. a minimum patentability standard.

To avoid confusion with the recent literature on the optimal breadth of patents, it should be stressed a minimum patentability standard and patent breadth are two quite different concepts. The latter determines the boundaries of the market for the patented good and its optimal size will depend crucially on the characteristics of demand for patented and non-patented goods. A minimum patentability standard refers ultimately to the minimum investment in R & D necessary for an invention to qualify for a patent, i.e. to the supply side of inventions. A fully-fledged model would encompass the optimal determination of both. As an example of the relationship between the two concepts, consider the case of R & D on genetic engineering leading to the invention of a DNA sequence with potentially beneficial effects. A high

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minimum patentability standard would take the form of requiring the would-be patentee to develop the initial idea into a well-defined product with specific beneficial properties as a precondition for patenting. Provided the minimum requirement was met, the Patent Office would then determine patent breadth, by establishing how different a DNA sequence would have to be in order not to infringe the original patent. Interestingly, national patent laws differ significantly on the scope of patents (with Japan granting much 'narrower' patents than the U.S.A.),¹ but not in the definition of minimum patentability standards, the vaguest hint of 'industrial applicability' (in the U.K.) or 'usefulness' (in the U.S.A.) being the necessary requirement for patentability.

It should be noted that with the exception of Wright (1983) who analyses patents within a prices-vs-quantities framework, all optimal patent models fail to provide a rationale for the very existence of patents, in so far as they assume perfect information. Gilbert and Shapiro (1990) and Klemperer (1990) sidestep the issue by assuming that the reward to be given to an inventor is exogenously determined. Tandon (1982) recognizes the superiority of lump-sums as a means for rewarding inventors but still proceeds to formulate a two-instrument patent mechanism which is informationally as demanding as a first-best lump-sum scheme. The present note still falls short of providing a rationale for patents in so far as it too assumes perfect information. However it does make a contribution to the field of optimal incentive design in that it characterizes the relative merits of alternative schemes, thereby providing a wider menu of choices for the modelling of the more realistic case of patenting under asymmetric information.

As a preview of the argument to be developed below, it is instructive to consider patent design as a two-player game and contrast it with a familiar duopoly game. Unlike a duopoly game, in which both control variables (e.g. prices, output levels) and the players' strategic roles (follower, leader) are exogenously determined, in the context of a patent game it is one of the players that sets the rules of the game. It is the Patent Office's job to determine how patents are applied for and granted. Thus a fully optimizing Patent Officer (P.O.) has to solve a double assignment problem, choosing both her role, i.e., either $F(ollower)$ or $L(eaders)$, and her control variables, i.e. either minimum patentability standard, $\sigma$, or patent life, $T$. All models of optimal patent life implicitly assume that welfare is always maximized under the game in which the P.O. leads and optimizes over $T$. However, it seems obvious that the solution of the above game selection exercise will in general depend on demand and technology conditions; and this is, in fact, the conclusion reached here – more specifically, it is shown by means of a very

¹See Klemperer (1990) and the references cited therein.
simple model that when innovations are not ‘difficult’ leadership is irrelevant and setting patentability standards is more efficient than setting patent life.

2. Patent games

The game in which the Innovator plays role $R$ and optimize over $z$, with the P.O. playing role $S$ and optimizing over $w$ will be referred to as a $(Rz, Sw)$ game. In order to quantify the notion of minimum patentability standard, consider a product that in absence of R&D investment is marginally unprofitable, i.e. $P(0) + \varepsilon = c$, where $P(Q)$ is the inverse market demand function, $c$ is marginal cost, and $\varepsilon$ is ‘small’. Define $\sigma$ as the extent of the innovation, i.e. the extent by which either $P(Q)$ is shifted outwards or $c$ is reduced. I shall take $\sigma$ as the patentability standard variable.

The present value of the Innovator’s per-period profits can be written as

$$\pi(\sigma, T) = \frac{1}{r} \int_0^T e^{-r t} G(\sigma) \, dt - R(\sigma)$$

where $G(\sigma)$ is gross per-period profit and $R(\sigma)$ is the Innovation Possibility Function that maps expenditure on R&D inputs, $R$, into the extent of the innovation, $\sigma$, $r$ is the social (and private) rate of discount, and $T$ is patent life. Assuming a distribution-neutral P.O., her objective is to maximize the present value of the sum of consumers’ surplus ($C(\sigma)$) and gross profits ($G(\sigma)$), net of R&D cost ($R(\sigma)$), i.e.

$$W(\sigma, T) = \int_0^T e^{-r t} [C(\sigma) + G(\sigma)] \, dt$$

$$+ \int_0^T e^{-r t} [C(\sigma) + G(\sigma) + WL(\sigma)] \, dt - R(\sigma), \text{ i.e.,}$$

$$W(\sigma, T) = \int_0^T e^{-r t} [C(\sigma) + G(\sigma)] \frac{1}{r} + WL(\sigma) \frac{e^{-r t}}{r} - R(\sigma), \tag{2}$$

where $WL(\sigma)$ is the Welfare Loss due to monopoly pricing.

The above implies that after the patent has expired the new technology becomes freely available and production of the new good continues under perfectly competitive conditions.
It is useful to write \( \tau = 1 - e^{-rT} \) and, noting that as \( T \) ranges from 0 to \( \infty \), \( \tau \) ranges from 0 to 1, to imagine that there is a patent life constraint \( \tau \leq 1 \). Excluding the double leadership case, there are in principle six games the P.O. can choose from, and these can be grouped in three classes: \{ (F\sigma, F\tau), (L\sigma, F\tau) \}, \{ (F\tau, F\sigma), (L\tau, F\sigma), (F\tau, L\sigma) \}, and \{ (F\sigma, L\tau) \}, with each class yielding a distinct equilibrium, as shown below.

**Proposition 1.** Irrespective of the Innovator’s strategic role, followership by the P.O. combined with optimization over \( \tau \) yields the economically trivial \((0,0)\) equilibrium (no innovation, zero patent life).

Assuming that the second-order condition for a maximum holds, in both the \((F\sigma, F\tau)\) and the \((L\sigma, F\tau)\) games the P.O.’s reaction function obtained from the first-order condition for the maximization of (2) is degenerate, i.e. \( \tau = 0 \).

**Proposition 2.** Irrespective of the Innovator’s strategic role, optimization over \( \sigma \) by the P.O. yields the \((\sigma^*, 1)\) equilibrium, i.e. patent life is infinite \((\varepsilon = 1)\) and the minimum patentability standard \( \sigma^* \) is determined by \( \tau^{PO}(\sigma^*) = 1 \), where \( \tau^{PO}(\sigma) \) is the P.O.’s reaction function.

Consider first the \((L\tau, F\sigma)\) game which, it may be noted, is the mirror-image of the traditional optimal patent-life model: whereas in the latter the P.O. plays the leader role, here she chooses to follow and, in contrast with the conventional model, allows the innovator to determine patent life.

The innovator-leader’s problem is:

\[
\max_{\tau \leq 1} \pi(\tau) = \frac{\tau}{r} G(\psi(\tau)) - R(\psi(\tau)),
\]

where \( \psi(\tau) = \tau^{PO^{-1}}(\sigma) \) and, from (2) the P.O.’s reaction function is\(^2\)

\[
\tau^{PO}(\sigma) = \frac{1}{L_\sigma} (C_\sigma + G_\sigma + W L_\sigma - rR_\sigma).
\]

It is simple to show that \( \pi_\tau(\tau) > 0 \), and thus the patent-life constraint is binding \((\tau = 1)\) and the minimum patentability standard \( \sigma^* \) is determined by \( \tau^{PO}(\sigma^*) = 1 \):

\[
\pi_\tau(\tau) = \frac{G}{r} + \psi_\tau(\tau) \left\{ \frac{\tau}{r} G_\sigma - R_\sigma \right\} > 0.
\]

\(^2\)In obvious notation, subscripts show differentiation.
Inequality (5) follows from the fact that the quantity in curly brackets equals 
\(-\frac{1}{r}\{C_s(\sigma)+(1-r)[WL_\sigma(\sigma)+G_\sigma(\sigma)]\} < 0\) and \(\psi_\sigma(\tau)\) is the reciprocal of the slope of the P.O.'s reaction functions, which from the second-order condition for the maximization of (2) is negative.

Under both the \((F_{\tau}, F_{\sigma})\) and \((F_{\tau}, L_{\sigma})\) games the Innovator's first-order condition \([1-r]G_\sigma=0\) yields a degenerate reaction function (i.e. the best-reply level of \(r\) is a constant) which coincides with the \(\tau=1\) line. Therefore both games yield the \((\sigma^*, 1)\) equilibrium, with infinite patent life and \(\sigma^*\) determined by the intersection of \(\tau=1\) and the P.O.'s reaction function (see point A, fig. 1).

Before turning to the welfare comparison among different patent regimes, it may be useful to provide an intuitive economic interpretation of the reaction function \(\tau'(\sigma)\). Player j's reaction function \(\tau'(\sigma)\) can be interpreted as an 'innovation contract' that specifies the minimum patentability standard \(\sigma\) required to be granted a patent term \(\tau\). In the standard game, the Innovator provides the P.O. with a menu of profit-maximizing \((\sigma, \tau)\) pairs and the latter selects the least inefficient combination. In the alternative game.
(Lτ, Fσ), the P.O. provides the Innovator with a menu of welfare-maximizing (σ, τ) pairs and the latter chooses the most profitable.³ It should be noted that although neither game describes current patent law, the proposed new game (Lτ, Fσ) is more likely to be implemented by a reformist Patent Office. The main limitation of the standard regime as a starting-point for patent reform is that it requires patent life to be industry-specific (innovation-specific, in fact). Anyone vaguely familiar with patent law and international patent treaties would dismiss as utterly unrealistic the notion of variable patent life as an implementable policy change. Conversely, the new game calls not for changes in patent life,⁴ but for industry-specific minimum patentability standards, which are more easily implementable within existing patent rules.⁵

So far we have shown that out of six possible patent games two, ((Fσ, Fτ) and (Lσ, Fτ)) yield the economically insignificant solution of no innovation, three – namely (Fτ, Lσ), (Fτ, Fσ), and (Lτ, Fσ) – are feasible and all sustain the same equilibrium, characterized by an infinite patent life and by a minimum patentability standard, σ*, lying on the P.O.'s reaction function.

The interesting question, of course, is whether the latter equilibrium can yield a higher level of welfare than the traditional version of the patent game, in which the P.O. acts as a leader and optimizes w.r.t. patent life τ (i.e. the (Fσ, Lτ) patent game). The main result of this Note is the following:

**Theorem.** The set of cost and demand parameters such that the (Fσ, Lτ) patent game yields lower welfare levels than under the alternative game (Lτ, Fσ) is non-empty.

**Proof.** As the second-order conditions for the maximization of (1) and (2) guarantees that the innovator's and the P.O.'s reaction functions are respectively upward- and downward-sloping, to prove the theorem it suffices to show that they cross at a point where τ>1, or, equivalently, that {τ^P0(σ*) = τ^I(σ) = 1} ⇒ (σ* > σ̅), as in fig. 1. This implies not only that the alternative patent game (Lτ, Fσ) performs better than the traditional (Fσ, Lτ) game whenever the latter calls for an infinite patent life, but also (deploying a continuity argument) that there exists a set of cost and demand parameters such that the two games yield the same level of welfare W(σ*, 1) = W(σ^*, 1), with 1 < 1, as shown in fig. 1. Therefore the (Lτ, Fσ) game performs strictly

³Notice that the information requirements of the two games are the same, for in both cases the P.O. has full information on the Innovator's technology and on demand.


⁵Interestingly the recent attempt by the European Commission to overhaul patent law in the biotechnology industry [see European Commission (1988)] does contain a clause (article 14) which implicitly establishes a link between patentability and the amount of R&D investment, thus approximating the optimal minimum patentability standard considered in the text.
better than the classic \((F\sigma, L\tau)\) game whenever demand and cost parameters are such that the latter game calls for an optimal patent life \(\tau^0, \hat{\tau} < \tau^0 \leq 1\).

The Innovator's reaction function is as follows:

\[
\tau^I(\sigma) = \hat{\tau} \frac{R_\sigma(\sigma)}{G_\sigma(\sigma)}.
\] (6)

Let \(\tau^{PO}(\sigma^*) = 1, C_\sigma(\sigma^*) + G_\sigma(\sigma^*) - rR_\sigma(\sigma^*) = 0\), then

\[
\tau^I(\sigma^*) = \frac{C_\sigma(\sigma^*) + G_\sigma(\sigma^*)}{G_\sigma(\sigma^*)} > 1.
\] (7)

Define \(\hat{\sigma}\) as \(\tau^I(\hat{\sigma}) = 1\), then, as \(\tau^I_\sigma(\sigma) > 0, \hat{\sigma} < \sigma^*\). Q.E.D.

2. An example

Resorting to specific cost and demand functional forms we can ascertain under what conditions which patent game performs better.

Let \(P(Q)\) be linear in output and \(R(\sigma)\) be iso-elastic:

\[
P(Q) = a - Q,
\] (8)

\[
R(\sigma) = \theta\sigma^{1/\alpha}.
\] (9)

In order to guarantee that in the absence of R & D investment production is (marginally) unprofitable, let the pre-innovation marginal cost be constant and equal to \(a\).

Using (8)–(9), the reaction functions (4) and (6) are as follows:

\[
\tau^{PO}(\sigma) = 4 - \frac{4\theta r}{\alpha} \sigma^{(1 - 2\alpha)/\alpha},
\] (10)

\[
\tau^I(\sigma) = \frac{2\theta r}{\alpha} \sigma^{(1 - 2\alpha)/\alpha}.
\] (11)

Under the \((F\sigma, L\tau)\) regime, in which the P.O. maximizes welfare w.r.t. \(\tau\), taking \(\tau^I(\sigma)\) as a constraint, the (unique) solution is:

\[
\hat{\tau} = \begin{cases} 
\frac{8\alpha}{1 + 4\alpha} & \text{for } \alpha < 1/4 \\
1 & \text{for } 1/4 \leq \alpha < 1/2,
\end{cases}
\]

\[
\hat{\sigma} = \begin{cases} 
\left(\frac{4\alpha^2}{(1 + 4\alpha)\theta r}\right)^{\alpha/(1 - 2\alpha)} & \text{for } \alpha < 1/4 \\
\left(\frac{\alpha}{2\theta r}\right)^{\alpha/(1 - 2\alpha)} & \text{for } 1/4 \leq \alpha < 1/2,
\end{cases}
\] (12)
where $\alpha < 1/2$ in order for the second-order condition to be satisfied.

If innovations are ‘easy’ (i.e. $1/4 \leq \alpha < 1/2$), granting of a patent is not an effective way of counter-balancing the output-restricting behaviour of an Innovator-Monopolist. The P.O. hits the $\tau \leq 1$ constraint when the marginal benefit of patent life extension is still positive. Indeed the Innovator is able to attain his first-best optimum and social welfare would be unaffected if patent term were self-administered by innovators. When innovations are ‘difficult’ ($\alpha < 1/4$) short-lived patents are an effective means to check the propensity to over-invest in R&D by an unregulated Innovator-Monopolist (assuming $\alpha = 0.1$, optimal patent life is less than 6 (3) years if $r = 10\%$ ($20\%$)). Somewhat surprisingly, given the functional forms (8)–(9), patents are more efficient in curbing potentially excessive innovation than in promoting it.

For the parameterization (8)–(9) the equilibrium minimum patentability standard under a $(LT, Fa)$ scheme, $\sigma^*$, turns out to be

$$\sigma^* = \left( \frac{3\alpha}{40r} \right)^{\alpha/(1 - 2\alpha)}.$$  \hspace{1cm} (13)

Computing the levels of welfare associated with the equilibrium of each of the two patent games, namely $W(\ell, \delta)$ and $W(\sigma^*, 1)$ for all values of $\alpha$ reveals that the alternative $(LT, Fa)$ game yields an improvement on the conventional $(Fa, LT)$ not only when innovations are easy (i.e. $1/4 \leq \alpha < 1/2$), but also when they are ‘not too difficult’ (i.e. $1/7 \leq \alpha < 1/4$).

This simple parameterization has the advantage of showing rather dramatically that the presence of a binding constraint on the control variable set by the leader may reverse the benefits of being a leader at all. When innovations are not ‘difficult’ the social benefit flowing from the ability to set a ‘high’ minimum patentability standard $\sigma^*$ more than offsets the cost of letting the innovator choose an infinite patent life.\(^6\)

\(^6\)It is interesting to note that, albeit for a smaller range of values of $\alpha$, the $(LT, Fa)$ game may outperform the standard game in which the P.O. leads, even if the latter is endowed with two instruments, patent life and a compulsory royalty rate, as in Tandon (1982).

**References**


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