THE CASE FOR PERMISSIVE PATENTS*

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Whatever its shortcomings in practice, until very recently patent law has sought to embody the winner-takes-all principle. This paper argues that it is both feasible and desirable to allow multiple prizes. Indeed, it is possible that complete abolition of the patent system, by raising the returns to late finishers, would yield an improvement on the current situation.

'A British court has just decreed that Genentech, a biotechnology company in California, cannot retain exclusive marketing rights in Britain for its heart product, TPA. The judge ruled that the terms of the patent were too broad (…) To stop others working (…) would stifle research and not be in the public interest. Genentech plans to appeal against the decision. If its patent were to stick, 19 companies would have to abandon their work on their TPAs.'

[The Economist, July 18, 1987]

1. Introduction

Recent advances in biotechnology have provoked debates on the proper scope of the patent system – reflected in the above quotation – which the theoretical literature, in so far as it fails to consider any intermediate solution between a strict first-past-the-post system and the complete abolition of patents, does not address. The problem is that, even though in practice late finishers generally obtain some positive payoff to their R&D effort, formal models of patent races typically assume that the winner takes all.1 This

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1Well known examples are Dasgupta and Stiglitz (1980), Loury (1979), Lee and Wilde (1980).
paper examines whether it is socially desirable to adopt policies which increase the extent to which the rewards to R&D are shared between the participating firms.

There is no doubt that policy can influence not merely the aggregate return to R&D but also its division. Suppose, at one extreme, that the patent system were abolished. Incentives to undertake R&D would still remain [see Plant (1934)]. Production lags, enhanced by industrial secrecy, give early inventors temporary market power before copies appear. Indeed, it is well documented that researchers sometimes fail to apply for patents on their discoveries even though they are available [see, e.g. the references in Levin (1986)]. Moreover, in the absence of patents, a genuine inventor who independently replicates a discovery, say a month after the first to find it, does benefit because even with two producers super-normal profits will generally be available. This feature is in contrast to the reward structure implicit in the traditional patent system in which the independent inventor who completes the research project a little late receives nothing (at least if patent life is of any length) just as does the plagiarist who simply seeks to copy the work of others.

Even without going to the extreme of abolishing the patent system it is possible to widen the distribution of returns to R&D. The narrower the interpretation of the protected idea the more scope for rivals to benefit from their own slight differentiated R&D. Even if discoveries are identical late finishers can be rewarded. In view of the administrative lag between application for and awarding of a patent (two years on average), a simple way of increasing the number of prizes is to grant a patent to all applications received up to the date of the award of a patent to the first claimant, with the provision that details of filed patents are kept secret until then.

Is there any merit in a losers-take-some reward structure? Given risk aversion it follows that, other things equal, sharing the rewards will enhance the attractiveness of R&D and so increase the flow of R&D. As demonstrated in appendix A, this may well be an important benefit, however, it is not the effect we focus on in the text, for risk neutrality is assumed. Even so, the risk reduction implicit in a reward structure that shares the returns may have real effects. With a conventional patent system, all that matters is getting home first and so risk-neutral firms engaged in a patent race have an incentive to gamble on a risky research strategy [see Klette and de Meza (1986a)]. Completing the course in the expected time virtually guarantees that the race will be lost, at least if there are many competitors. When all that matters is winning, it is better to select a bold strategy that yields the possibility of a very fast time even if there is a even greater chance of complete failure. It makes no matter whether you are second or last. In a R&D context, the consequence is that R&D strategies turn out to be excessively risky from a social viewpoint. Although inventing a week before a
rival has a high private payoff, the social advantage is slight. Introducing multiple prizes diminishes the cost of not being first and therefore leads to a socially preferred choice of research strategy. However, this potential benefit is also excluded from the present analysis, for firms are assumed to be unable to influence the riskiness of their R&D strategies.

The concern of this paper is with the trade-off implicit in the fact that, while allowing more than one inventor to profit from R&D leads to non-cooperative behaviour which diminishes the expected return to R&D, given the volume of R&D, competition between inventors benefits the users of the invention. The issue is whether it is better to encourage competition in the discovery or the dissemination of new ideas. From this perspective, whether or not it is desirable to introduce multiple prizes therefore seems ambiguous and, broadly speaking, that is the conclusion our analysis leads to. If there are constant returns in production and patent life can be set optimally, there is a strong presumption that multiple prizes are strictly preferable. The presence of economies of scale weakens this tendency. If the choice of patent life is effectively beyond the control of the authorities, as will subsequently be argued to be sometimes the case, then even with constant returns it is ambiguous whether multiple prizes are desirable.

The model developed here is highly stylized. Our purpose is to suggest that the winner-takes-all features of the conventional patent system is not necessarily the best possible arrangement. In practice the existing patent system often does give some reward to late finishers and thus lies between the two polar cases we analyse. The purpose of the paper is to identify whether it is desirable to introduce this feature deliberately. Our results are likely to be robust, but it must be recognized that many practical details will be treated cursorily. Nevertheless, we believe that our analysis makes the case that it is worth taking alternatives to the present patent system seriously.

2. The model

The general structure of the model follows that of Dasgupta and Stiglitz (1980). Risk-neutral firms each spend an amount $x$ on R&D at time 0 and this yields a known probability of inventing at each subsequent moment. For most of their analysis Dasgupta and Stiglitz specify a Poisson distribution of invention times but for our purpose there is no need to specialise the density function. Firms follow independent R&D strategies and seek a particular new product. At moments at which there is but a single producer it earns monopoly profits of $\pi_1$. When there are $n > 1$ producers collusion is

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2Dasgupta and Stiglitz show that even if $x$ is variable, with large numbers of entrants, its equilibrium value is always the same.
imperfect and each earns gross profits of $\pi_n$, with $n\pi_n < \pi_1$. As we assume that R&D is aimed at producing a new product (i.e. a product which prior to invention cannot be sold at a profitable price) and that all potential producers are equally efficient, it is easy to see that a successful multiplant inventor will never choose to licence other firms to produce the new product.

Two patent regimes will be considered. Under the strict regime the first to invent receives a patent lasting for $T_s$ years. During this period no other firm receives a patent and so cannot produce. When the patent expires there is freedom of entry into the industry. Under the alternative permissive regime, patents are awarded to all genuine inventors (i.e. non-plagiarists) as long as they apply within $T'$ years of the first claimant. This regime could be implemented through the open-registry scheme discussed in the introduction.

It must be admitted that there is an incentive for one inventor to buy up all patents issued and so extract monopoly profits; however, with small numbers negotiations may be difficult and costly. Moreover, the analysis of Salant, Switzer and Reynolds (1983) shows that even ignoring transaction costs the strategy would be unprofitable unless a sufficiently high proportion of patents are secured. In any case it would be easy for antitrust authorities to enforce a restriction that a firm cannot own two or more patents on the same product/process. So we do not assume that monopolisation is inevitable.

A permissive regime with patent life $T_p$ allows for multiple producers even during the life of the first patent granted. Once the first patent expires the idea is public property and, because of the long lead time, new production can start quickly, rendering all other patents economically worthless.

**Proposition 1.** If patent life is a policy variable the permissive regime is welfare superior to the strict regime if there are constant returns to scale in the production of the new good and market demand is linear or of constant elasticity.

**Proof.** The general form of the proof is to show that if $T_s$ is the optimal patent life in the strict regime then in a permissive regime there exists a patent life $T_p > T_s$ which attracts the same number of entrants to the race and yields greater social benefits. The optimal $T_p$ must therefore definitely be socially preferred to the best outcome in the strict regime.

First, some notation. Let there be $n$ entrants to the R&D stage under the strict regime and write the density function that the first discovery occurs $t_1$, periods after time 0, the second after $t_2$ periods and so on as $P(t_1, \ldots, t_n)$. Under a strict regime (using a formulation that appears cumbersome but is subsequently useful) a free-entry equilibrium satisfies

$$E\{P(t_1, \ldots, t_n)\pi_1 g(t_1)\} - nx = 0,$$  \hspace{1cm} (1)
where
\[ g(t_i) = \frac{1}{r} \left[ e^{-r t_i} - e^{-r (t_i + T_p)} \right], \quad i = 1, \]
\[ = 0. \quad \forall i > 1. \]

\( r \) is the discount rate and \( \pi_1 \) is instantaneous monopoly profit.

Consider now the permissive regime. Since there is normally a positive probability that the first inventor will face competition within the life of its patent and assuming that in that event collusion is imperfect, it follows that with no change in patent life or the number of entrants, expected profits will be lower. Hence if the number of inventors is to be maintained at \( n \) it follows that patent life must be increased, say to \( T_p \). It is not obvious that there will exist a \( T_p \) sufficiently large to have this effect but in appendix B it is shown that this is not a problem. Here we simply assume that a \( T_p \) with the required property does exist. Instantaneous gross per-firm profits when \( n \) firms are active are \( \pi_n \) and \( n \pi_n < (n - 1) \pi_{n-1} \).

If \( n \) firms enter the R&D race, then under the permissive regime

\[ E \left\{ P(t_1, \ldots, t_n) \sum_{i=1}^{n} i \pi_i f(t_i) \right\} - nx = 0, \tag{2} \]

where
\[ f(t_i) = \frac{1}{r} \left[ e^{-r t_i} - e^{-r (t_i + t_{i+1})} \right], \quad t_{i+1} > t_i, \quad i \geq 1, \]
\[ = \frac{1}{r} \left[ e^{-r t_i} - e^{-r (t_i + T_p)} \right], \quad t_{i+1} > t_i + T_p > t_i, \quad i \geq 1, \]
\[ = 0, \quad \forall i > 1. \]

From (1) and (2)

\[ E \left\{ P(t_1, \ldots, t_n) \left[ f(t_1) - g(t_1) \right] + \sum_{i=2}^{n} f(t_i) i \pi_i / \pi_1 \right\} = 0, \tag{3} \]

Now consider the welfare implications of the switch in regimes. Aggregate social efficiency, net of R&D costs, at each moment is the sum of consumer surplus plus industry profits. This is clearly maximized when price equals
marginal cost. Actual welfare may therefore be written as the difference between this maximum level of welfare, \( W^* \), and \( D_i \), the temporary deadweight loss incurred for the duration of the patent. It follows that

\[
E\{W_s\} = E\{P(t_1, \ldots, t_n)[(W^*/r)e^{-nt_1} - D_1 g(t_1)]\} - nx, \tag{4}
\]

\[
E\{W^p\} = E\{P(t_1, \ldots, t_n)\left[ (W^*/r)e^{-nt_1} - \sum_{i=1}^{n} D_i f(t_i) \right]\} - nx. \tag{5}
\]

These equations embody the fact that when the first patent expires deadweight loss is zero if at least one firm has invented. Following the same procedure as before, a switch from the strict to the permissive regime yields a welfare gain if

\[
E\left\{ P(t_1, \ldots, t_n) \left\{ [f(t_1) - g(t_1)] + \sum_{i=2}^{n} f(t_i)D_i/D_1 \right\} \right\} < 0. \tag{6}
\]

From (3) and (6) it is evident that the switch raises welfare if

\[
i\pi_i/\pi_1 > D_i/D_1, \quad \forall i \leq n. \tag{7}
\]

Appendix C shows this condition is satisfied if demand is linear or of constant elasticity.\(^3\)

Although we have not characterized optimal life under a permissive regime, we have identified a policy which is welfare superior to the optimal strict patent.

Proposition 1 has been proved for specific functional forms, albeit the most frequently assumed in practice. Some specialisation is required because knowledge of merely local properties of the demand curve is insufficient to evaluate deadweight losses. However, an indication of the robustness of the result obtained can be obtained by treating \( n \) as a continuous variable.

In appendix D it is easily established that under Cournot assumptions

\[
\frac{d \log n \pi_i}{dn} > \frac{d \log D(n)}{dn} \quad \text{iff} \quad 1 - n + \frac{[P(n) - c]Q(n)}{D(n)} > 0. \tag{8}
\]

When \( n = 1 \) this condition is always satisfied and as (8) is the continuous version of (7) this suggests that if the rules of the permissive regime restrict the number of patentees to two at most, then it must be preferable to a strict regime. However, as the change from \( n = 1 \) to \( n = 2 \) is not small, (8) could

\(^3\)In fact with these functions the result holds for any kind of imperfect collusion \((n\pi_n \text{ declining in } n)\), not merely with Cournot competition.
turn out to be a poor approximation of (7). However, it can be shown in the continuous case only exceptionally will Proposition 1 fail.

Notice that, as the deadweight loss triangle $D$ is less than the rectangular area $\Delta Q(P-c)$, where $\Delta Q$ is the increase in market demand when price falls from $P(n)$ to $c$, inequality (8) can be written as

$$1 - n + \lambda Q/\Delta Q > 0,$$

where $\lambda \equiv (P-c)\Delta Q/D > 1$.

Under Cournot assumptions $P(n)-c$ equals $P(n)/\varepsilon n$, where $\varepsilon$ is the point elasticity of demand evaluated at $P(n)$.

Defining the arc-elasticity of demand $\varepsilon^*$ as $(\Delta Q/\Delta P)(P(n)/Q(n))$, where $\Delta P = P(n) - c$, it follows that (9) implies

$$\varepsilon > (n-1)/\lambda n.$$

So, if (8) is to fail

$$1 > (n-1)/\lambda n > \varepsilon^*,$$

i.e. $\varepsilon^* > \varepsilon$; this in turn requires that over the relevant range the elasticity $\varepsilon$ be increasing in $Q$. This imposes a severe restriction on the curvature of the inverse demand curve $P(Q)$; in fact it rules out concave, linear and constant-elasticity demand curves and requires $P(Q)$ to be strongly convex.

However, if $P(Q)$ is strongly convex, the deadweight loss $D$ is significantly less than half $(P-c)\Delta Q$. Hence $\lambda$ is certainly greater than 2. To violate (8) it is thus required that $\varepsilon^* \geq (2n/(n-1))\varepsilon$.

This is the basis for claiming that only if the point elasticity of demand rises very rapidly as price falls will a strict regime be preferable to a permissive patent regime.

**Proposition 2.** A strict regime may be superior to a permissive regime if economies of scale in production are sufficiently great.

Now put everything in reverse. Let $T_s^*$ be the optimal patent life under the permissive regime. Choose $T_s^*$ to yield the same number of firms. Following the same procedure that leads to (7), the strict regime is certainly welfare superior if

$^4$Straightforward differentiation reveals that in order for $d\varepsilon/dP$ to be negative the elasticity of the slope of the inverse demand curve (i.e. $-(P'(Q)/P'(Q)Q)$) must exceed $(1+\varepsilon)/\varepsilon$, i.e., $P(Q)$ has to be 'strongly' convex.
where $D_i^* \geq 0$ is the difference between net social surplus when there is a free-entry equilibrium in the producing industry (i.e. the equilibrium when knowledge is public) and realized aggregate benefit when there are only $i$ firms producing. Notice that, because of economies of scale, it is possible that a free-entry equilibrium sustains too many firms and hence $D_i^* < 0$.

Appendix E shows that with linear demand curves and a fixed production cost, inequality (12) may be satisfied. The economic intuition underlying Proposition 2 is simply that in the presence of economies of scale production efficiency benefits from concentration and this may more than offset the losses from reduced competition.

In the Dasgupta–Stiglitz model all R&D costs are sunk at the start of the programme. Even if the instantaneous probability of discovery depends only on current R&D expenditure, our results go through. Nevertheless, it might be argued that a strict patent system coupled with compulsory licensing, by avoiding the R&D costs incurred by the late finishers while the registry is open, may be even better than a permissive patent regime. However, a permissive scheme does not suffer from some of the problems that occur with licensing:

(i) by sharing rewards, the permissive patent cuts the real costs of risks if, as is almost certainly true, inventors are risk-averse. This benefit would not be captured by a compulsory licence scheme which still preserves the winner-takes-all feature;

(ii) a compulsory license scheme gives an inventor an incentive not to apply for a patent but instead rely on secrecy. Resources will be expended to try to maintain secrecy and, in any case, the period of protection afforded by secrecy may be inadequate from a social viewpoint;

(iii) a compulsory license disclosing the barest details of a new process/product may sometimes not be of much value to a user if the inventor is not willing to cooperate by providing ancillary information;

(iv) compulsory licenses will entail resource costs in monitoring revenues, outputs and so on to pay the fees accurately;

(v) in practice, late finishers will not normally come up with identical ideas, even though under the existing rules these may not be sufficiently novel to be granted a patent. Still, the additional variety may well be enough to justify the R&D costs expended during the possibly short time the registry remains open.

It has been assumed so far that demand for the new product is stable through time. But it is often the case that succeeding waves of technical change render existing discoveries obsolete. Thus many firms taking out patents have little expectation that the invention will be of commercial value
for the full legal life of the patent. In effect, the life of patents is constrained by economic not legal factors. When there is 'water in the patent' it follows that it will be impossible to set $T_p > T_r$. Both regimes must have equal patent life. Of course a change in regime may affect the date at which the next generation of products is expected to appear, but this complication will be ignored. There is another reason for interest in the case of equal patent life. As argued in the introduction and conclusion, abolition of the patent system results in an environment similar to that of the permissive regime albeit with, in effect, a short $T_p$. If we show that the welfare effects of a switch in regimes are ambiguous when $T_p = T_r$, this will remain true for some $T_p < T_r$.

**Proposition 3.** If patent life is the same under a permissive and a strict regime then, even with constant returns in production, it is ambiguous which is socially preferable.

In order to prove Proposition 3 it is sufficient to use a simpler version of the model sketched above. Upon payment of a research fee, $x$, a firm is given a lottery ticket that yields at some future moment a particular product-innovation with probability $p$. If the firm does not invent at that time, it never will. Under the strict regime the (single) patentee reaps the reward flowing from his invention in the form of gross monopoly profits, $\pi_1$. In the event of $m$ firms 'striking gold', each is awarded $\pi_1$ with probability $1/m$. If there are $N$ entrants and entry is free, the following zero expected profit condition will hold:

$$E^* \{ \pi \} = \frac{N-1}{i} \left( \begin{array}{c} N-1 \cr i \end{array} \right) p^{i+1} q^{N-(i+1)} \pi_1 \tau/(i+1) - x = 0, \quad p + q = 1,$$

where $\tau (= (1 - e^{-rT})/r)$ is the discount factor and $T$ is patent life.

Under the permissive regime, as all successful inventors are granted a patent, gross profits will depend on the number of successful inventions. Once again it is assumed that collusion is not feasible and that, if there are multiple patentees, they will engage in a Cournot game. With a linear inverse market demand curve and constant marginal costs, it is easy to check that the gross profit accruing to each of $i$ successful inventors is given by

$$\pi_i = 4\pi_1/(i+1)^2.$$

With free entry and $n$ entrants, the zero expected profits condition ensures that

$$E^* \{ \pi \} \equiv \sum_{i=0}^{n-1} \left( \begin{array}{c} n-1 \cr i \end{array} \right) p^{i+1} q^{n-(i+1)} \pi_{i+1} \tau - x = 0.$$
Under the stated assumptions, if \( i \) firms are active then aggregate deadweight loss, \( L_i \), is given by

\[
L_i = \frac{W^*}{(i + 1)^2}, \quad W^* = \max_{q} \left[ U(Q) - cQ \right].
\]

(16)

Net social welfare under the strict and the permissive regime can be written respectively as

\[
E\{W^S\} \equiv (1 - q^n)[(W^*/r) - (W^*/4)r] - N x,
\]

(17)

\[
E\{W^P\} \equiv (1 - q^n)(W^*/r) - \sum_{i=1}^{n} \binom{n}{i} p^i q^{n-i} W^*/(1+i)^2 - nx.
\]

(18)

By comparing (17) and (18) it can be established that

**Proposition 3.1.** With no uncertainty (i.e. \( p = 1 \)) the permissive regime is socially preferable.

In fact it is trivial to verify that under the permissive regime both aggregate deadweight losses and aggregate research expenditure are smaller (since \( n < N \)).

**Proposition 3.2.** The strict regime may be socially preferable to the permissive regime if inventions are 'difficult' (i.e. \( p \) is 'small').

The reason why Proposition 3.2 holds is easy to appreciate: a feature of the strict regime is that a free-entry equilibrium sustains more firms than a permissive regime (\( n < N \)); when inventing is 'difficult' the social benefit flowing from the additional entrants in the form of an increased probability of having at least one successful invention may exceed the double disadvantage of the strict regime, namely the higher research expenditure and the lower gross social surplus if an invention does occur. Of course Proposition 3.2 holds trivially if \( n < 1 \) and \( N \geq 1 \).

Consider the following example: let \( p = 0.1 \) and let research fees be such that the strict regime sustains precisely two entrants, i.e., setting \( W^* = 1 \) by a suitable choice of units, \( x = 0.0475r \).

It is easy to check that under the permissive regime only one firm will enter making (super-normal) profits amounting to 0.0025\( r \). Using (17) and (18) we can compute net social welfare under the two regimes:

\[
E\{W^S\} = (1 - 0.92)[(1/r) - (r/4)] - 2x,
\]
\[
E\{W^p\} = 0.1[(1/r)-(\tau/4)] - x + \text{supernormal profits.}
\]

It is easy to confirm that \(E\{W^p\} > E\{W^p\}\) since \((1/r) \geq \tau\).

3. Conclusion

The debate over the social net benefits of the patent system, in focusing on the stark alternative between a no-patent system (which allows perfect free-riding) and a 'strict' patent regime (which prevents genuine but late inventors from benefitting from their own R&D investment), has ignored the mid-way option of a 'permissive' regime. Such a system excludes true free-riders (i.e. those who have not invested in R&D) but does not penalize genuine inventors for not arriving first.

The permissive regime could be instituted by changing the rules by which patents are awarded. Taking into account that there exists an administrative lag between filing for a patent and obtaining it, a simple way of implementing a permissive regime would entail accepting all applications up to the date of the award of a patent to the earliest inventor of a given class of new products/processes. Of course, it must be assumed that, as under the current (strict) regime, the Patent Office will not divulge the technical details of patents before they are awarded even if under the permissive regime there would exist an incentive to bribe patent officers.

A no-patent regime can be interpreted as a permissive regime with patent lives shorter than production and imitation lags. Instead of disclosing early in return for a patent, inventors would minimize the flow of pre-production information and potential free-riders would have to wait until the product appear before copying it. This would give the initial inventors an interval in which to reap supra-normal profits and, more importantly, returns would not be limited to the first past the post. The length of time that inventors enjoyed protection from free-riding would probably be shorter than at present, but since it is already quite common for firms not to avail themselves of patents, this would not necessarily be a crippling blow. Of course, there are some products for which a patent remains valuable for the whole of its legal life. In such cases, the effective shortening of protection afforded to the inventor would be important and may more than offset the advantages of sharing the returns. However, it is worth noting that inventions that are of durable economic value are likely to be major inventions in the sense of generating large cost falls or significantly improved products. It is precisely such inventions that Klette and de Meza (1986b) show should anyway be rewarded with the shortest patents.

In the past, serious economic cases have been made that patents should be abolished, because they over-reward the inventor [e.g. Plant (1934)]. What
does not seem to have been appreciated is that abolishing patents tends to increase the rewards to coming second or third.

Although no unequivocal case can be made, we hope we have offered a persuasive argument that, for reasons that have previously been neglected, a 'permissive' patent regime, or even a no-patent regime, deserves serious consideration for, under reasonable assumptions, it may be socially preferable to what has traditionally been seen as the ideal of a 'strict' regime.

Appendix A

In this appendix we show that under risk aversion a relaxation of the strict patent regime which allows the first two inventors to be awarded a patent may yield a larger net social benefit.

In order to focus on risk aversion we shall ignore all other potential benefits flowing from a permissive patent regime examined in the text and assume that if two or more firms succeed at the invention stage the two eventual patentees share monopoly profits $\pi$.

Upon payment of a research fee $\phi$, each firm acquires a lottery ticket yielding a given new product with probability $p$. Let $u(y-\phi)$ be each entrant's utility function, where $y$ is the gross reward of the game, which, of course, depends on the nature of the patent regime: under a strict, first-pass-the-post, regime $y=\pi$ whereas under a more permissive two-patent regime if two firms succeed each receives $y=\pi/2$. We normalize $u(\cdot)$ so that $u(0)=1$.

Under a strict patent regime, assuming that if $k$ firms succeed each has a $1/k$ chance of reaping $\pi$, the equilibrium number of entrants $n$ will satisfy the following condition

$$u(n-q^n/n=1, \quad q=1-p. \quad (A.1)$$

Under a permissive two-patent regime, the equilibrium number of entrants $N$ is determined by the following:

$$pq^{N-1}u(\pi-\phi)+\left[p\sum_{i=1}^{N-1}\left(\binom{N-1}{i}p^{i}q^{N-1-i}2/(i+1)\right)u(\pi/2-\phi)=1, \quad (A.2)\right.$$

where

$$p\left(\binom{N-1}{i}\right)\pi^{i}q^{N-1-i}$$

is the probability that a successful firm will be faced with other $i$ successful
inventors and $2/(i+1)$ is the probability of being awarded one of the two available patents.

For future reference, notice that

$$p \sum_{i=1}^{N-1} \binom{N-1}{i} p^i q^{N-1-i} 2/(i+1) = 2(1-q^n)/N - 2pq^{N-1}.$$ (A.3)

If a permissive two-patent regime equilibrium sustains more firms than under a strict regime, then there will be a net social gain flowing from the higher probability of a discovery being made at all.

The following simple example shows that under risk aversion $N$ may indeed exceed $n$.

Suppose that $n=2$; then, from A.1 and A.2, a two-patent regime will produce a $50\%$ increase in the number of entrants (i.e. $N = 3$) provided that

$$pq^2u(\pi - \phi) + \left[ \frac{2}{3} (1-q^3) - 2pq^2 \right] u(\pi/2 - \phi) \geq [(1-q^3)/2] u(\pi - \phi).$$ (A.4)

It is simple to confirm that A.4 will hold iff

$$\frac{u(\pi/2 - \phi)}{u(\pi - \phi)} \geq \frac{3}{4}.$$ (A.5)

For appropriate values of $\pi$ and $\phi$ A.5 will be satisfied if the income elasticity of the utility function is less than $1/3$. Although the less risk averse inventors are the less dramatic the effect on entry, the permissive regime always encourages entry.

**Appendix B**

In the proof of Proposition 1 it is assumed to be possible to find a $T_p$ which induces as much entry as does $T_r$. This requires that $T_r$ is finite. It is easily shown that it normally will be. The demonstration that follows is for a zero discount rate, however this is no problem because it can be shown that $T_r$ is decreasing in $r$.\(^6\) Let the expected date of first discovery be $t = t(n)$. Granted that freedom of entry always results in zero expected profits for the firms, the social problem is to minimize the net loss to consumers from

\(^5\) Notice that A.4 holds for any $p > 0$; however this result does not extend to all $n$ and $N$.

\(^6\) There is an error in Dasgupta and Stiglitz's (1980) formulation of the optimal patent life problem. This is corrected in Klette and de Meza (1986a) who show $T_r$ to be a decreasing function of $r$. 


delaying the cheap availability of the invention. The problem is thus to minimize

\[ S = tS_0 + T_1S_1, \]  

(B.1)

where \( S_0 \) is the gain in consumer surplus when the new good is introduced at a price equal to marginal cost as opposed to \( S_1 \) which is the gain in consumer surplus when price falls from the monopoly level to marginal cost. Minimizing \( S \) requires

\[ \frac{\partial S}{\partial T_s} = S_1 + S_0 \frac{dt}{dn} \frac{dn}{dT_s} = 0. \]  

(B.2)

Now, entry is determined by the zero profit condition

\[ \frac{\pi_1 T_s}{n} - x = 0. \]  

(B.3)

Eq. (B.3) implies that for some finite patent lives R&D costs are sufficiently low to induce multiple entry. However the Dasgupta and Stiglitz approach makes no sense unless this is true.

Differentiating B.3

\[ \frac{dn}{dT_s} = \frac{n}{T_s} = \frac{\pi_1}{x}. \]  

(B.4)

From B.2 and B.4, at an optimum

\[ S_1 + S_0 \frac{dt}{dn} \frac{\pi_1}{x} = 0. \]  

(B.5)

However, when \( T_s \) is large and hence so is \( n \), \( dt/dn \) will be a negative number of small absolute value – when there are already many firms seeking to invent entry of another one cannot advance the expected date of first discovery by much. Thus for \( n \) large the LHS of B.5 must be positive. For B.5 to hold patent life must be lowered thereby raising \( dt/dn \) and reducing the aggregate deadweight loss. Optimal patent life is finite under a strict regime.

Granted that \( T_s \) is not infinite, it is still not certain that even an infinite \( T_p \) will induce the entry of \( n \) firms under the permissive regime. But instead of allowing an unlimited number of patents, if the reward was limited to, say, the first two inventors to succeed, it is virtually certain that if \( T_s \) induces \( n \)
firms then, except in the extreme case of Bertrand competition and perfect substitutes, duopoly profits would not be so low as to preclude \( n \) firms with a sufficiently large \( T_p \). This is of relevance because Proposition 1 would hold even if permissive is interpreted as two firms rather than one.

Appendix C

The iso-elastic market demand case

In obvious notation, inverse market demand is written as

\[
p = \left( \sum_{i=1}^{n} q_i \right)^{-1/\varepsilon}.
\]  

Let \( W_m \) be social welfare, net of production costs (but gross of research costs), when \( m \) oligopolists are active, i.e.

\[
W_m = \left[ c/(\varepsilon - 1) \right] \left( \sum_{i=1}^{m} \hat{q}_i \right)^{(c-1)/\varepsilon} - c \sum_{i=1}^{m} \hat{q}_i.
\]  

where \( \hat{q}_i \) is the profit-maximizing level of output of firm \( i \) in an \( m \)-firm Cournot oligopoly and \( c \) is marginal and average cost, i.e.

\[
\pi_i(\hat{q}_i) = \max_{q_i} \left\{ q_i \left[ q_i + \sum_{j \neq i} q_j \right] - cq_i \right\}.
\]  

It is simple to confirm that in a symmetric equilibrium

\[
\hat{q} = \left[ mce/(m\varepsilon - 1) \right]^{-\varepsilon/m} \quad \text{and} \quad \pi_m(\hat{q}) = \left[ mce/(m\varepsilon - 1) \right]^{1-\varepsilon/\varepsilon}.
\]  

Define \( W^* \) as the first-best level of gross social welfare, i.e.

\[
W^* = \max_{Q} \left\{ (\varepsilon - 1)/\varepsilon \right\} Q^{(\varepsilon - 1)/\varepsilon} - cQ \right\} = c^{1-\varepsilon}/(\varepsilon - 1).
\]  

Finally, define \( D_m \) as the deadweight loss associated with an \( m \)-firm oligopoly as compared with the first-best level of social welfare:

\[
D_m = W^* - W_m = \left[ 1/(\varepsilon - 1) \right] c^{1-\varepsilon} \{ 1 - [(m\varepsilon + \varepsilon - 1)/m\varepsilon] [(m\varepsilon - 1)/m\varepsilon]^{\varepsilon-1} \}.
\]  

For Proposition 1 to hold, it has to be shown that
\[ \frac{m \pi_m}{\pi_1} > \frac{D_m}{D_1}, \quad (C.7) \]
i.e., using C.4 and C.6
\[ \frac{[(me-1)/me]^{\epsilon-1}/m}{[(e-1)/e]^{\epsilon-1}/m} > \frac{\{1 - [(me-1)/me]^{\epsilon-1}/m\} - [(me-1)/me]^{\epsilon}}{1 - [(e-1)/e]^{\epsilon-1}/m - [(e-1)/e]^{\epsilon}}, \quad (C.8) \]
which holds for \( \forall m \geq 2 \). \qquad Q.E.D.

The linear demand case

Using the same notation as above it is easy to confirm that if the inverse market demand is linear, i.e.
\[ p = a - \sum_{i=1}^{m} q_i, \quad (C.9) \]
then Proposition 1 holds because
\[ m \pi_m/\pi_1 = \frac{4m}{(m+1)^2} > \frac{4}{(m+1)^2} = \frac{D_m}{D_1}. \]

Appendix D

Write industry profit as
\[ n \pi_n \equiv \pi(n) \equiv (P(n) - c)Q(n), \quad (D.1) \]
where \( Q(n) \) is total output. Hence
\[ \frac{d \log \pi(n)}{dn} = \left( 1 + \frac{dP}{dQ} \frac{Q}{P-c} \right) \frac{dQ}{dn}, \quad (D.2) \]
\[ D \equiv \int_{c}^{P} Q(z) \, dz - (P-c)Q, \quad (D.3) \]
and thus
\[ \frac{dD}{dP} = -(P-c) \frac{dQ}{dP}, \quad (D.4) \]
\[ \frac{d \log D}{dn} = \frac{(P-c)}{D} Q \frac{dP}{dn} \frac{dQ}{P} \frac{dP}{Q}. \quad (D.5) \]

But as
\[ \frac{d \log P}{dn} = \frac{dP}{dQ} \frac{Q}{P} \frac{dQ}{dn}, \quad (D.6) \]
\[ : \frac{d \log \pi(n)}{dn} > \frac{d \log D}{dn} \quad \text{iff} \quad 1 + \frac{dP}{dQ} \frac{Q}{P-c} + \frac{(P-c)}{D} Q > 0.\quad (D.7) \]
Eq. (8) of the text follows from (D.7), recalling that at a Cournot equilibrium
\[
\frac{dP}{dQ} \frac{Q}{P-c} = n.
\]

Appendix E

An example that satisfies inequality (12)

Suppose that \( T_p \) is such that only two firms enter the race \( (n=2) \) and that a free-entry equilibrium would sustain three firms if R&D were free. It is readily calculated that in the linear demand case fixed costs must amount to \( \pi_1^G/4 \), where \( \pi_1^G \) are monopoly profits gross of fixed costs. It is easy to verify that net monopoly and duopoly profits are respectively \( \pi_1 = (3/4)\pi_1^G \) and \( \pi_2 = (7/36)\pi_1^G \). The deadweight losses associated with \( \pi_1 \) and \( \pi_2 \) (taking as a benchmark the free-entry equilibrium) are given by
\[
D_1^* = -\frac{3}{8} \pi_1^G, \quad D_2^* = -\frac{17}{72} \pi_1^G.
\]

It follows that
\[
\frac{2\pi_2}{\pi_1} > \frac{14}{27} \frac{D_2^*}{D_1^*} = \frac{17}{27},
\]
as required for (12) to hold.

References


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